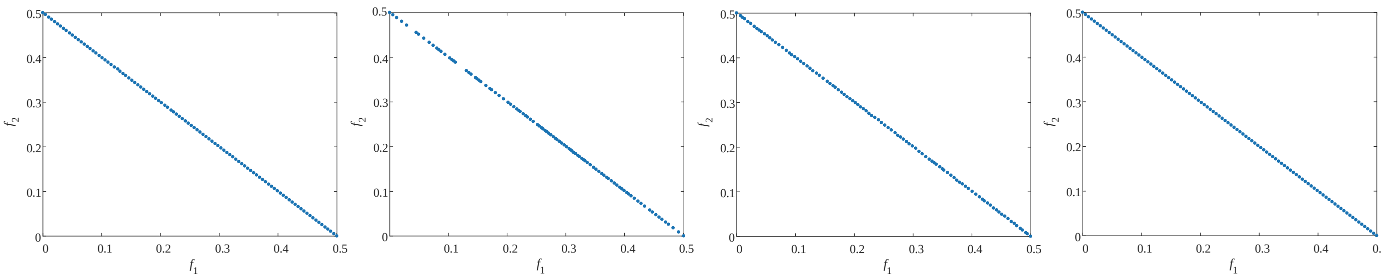
# The Supplement of the Paper "A Weight Adaptation Trigger Mechanism in Decomposition-based Evolutionary Multi-Objective Optimisation"

## I. Comparison with three non-decompositional multi-objective evolutionary algorithms

Table I and Table II show the IGD and HV results regarding the mean and standard deviation (SD) values of ATM-MOEA/D and the other non-decompositional peer MOEAs: AR-MOEA, VaEA, and BCE-IBEA. The better mean for each problem was highlighted in boldface. To have statistically sound conclusions, the Wilcoxon's rank sum test at a 0.05 significance level was used to test the significance of the differences between the results obtained by ATM-MOEA/D and the four peer algorithms. The tables show that ATM-MOEA/D ranks the best overall compared with the three peer algorithms, both on IGD and HV. For IGD, ATM-MOEA/D outperforms AR-MOEA, VaEA, and BCE-IBEA on 31, 34, and 25 out of the 36 test problems, respectively. In contrast, AR-MOEA, VaEA, and BCE-IBEA win only on 3, 2, and 5 test problems. For HV, ATM-MOEA/D outperforms AR-MOEA, VaEA, and BCE-IBEA on 28, 29, and 22 test problems, respectively. In contrast, AR-MOEA, VaEA, and BCE-IBEA win only on 2, 4, and 9 test problems.

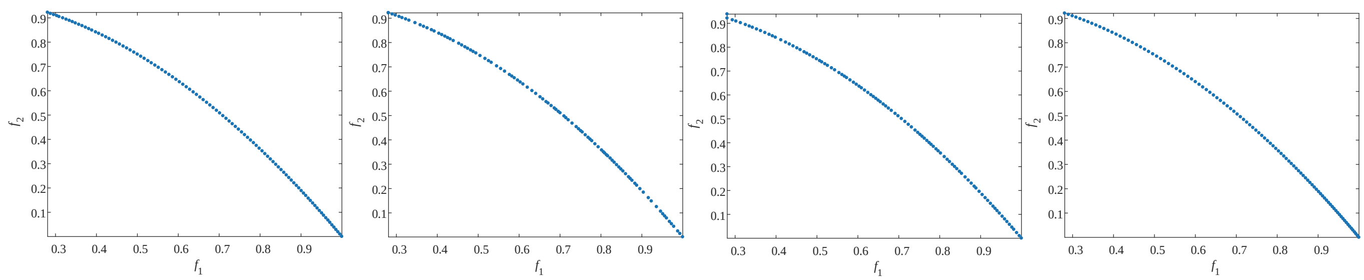
Figures 1–4 illustrate the final solution sets obtained by the algorithms on the representative problems with regular Pareto fronts. As shown in the figures, ATM-MOEA/D and AR-MOEA outperform the other algorithms on 2-objective problems (DTLZ1, ZDT6) and 3-objective problems (DTLZ1, DTLZ2) with its solutions being well distributed on a line or a face. AR-MOEA is an indicator-based EMO, AR-MOEA not only utilises reference points uniformly sampled from a unit hyperplane but also adaptively adjusts the distribution of reference points based on the contribution of candidate solutions in an external archive in terms of IGD-NS, thus the algorithm is effective for problems with a regular Pareto front. The solutions of the other two peer algorithms are obviously not as uniform as those of ATM-MOEA/D and AR-MOEA.

We also selected 26 test problems with an irregular Pareto front and evaluated the performance of the five algorithms using two metrics: IGD and HV. The results are presented in Tables I and II. In terms of IGD, ATM-MOEA/D outperforms AR-MOEA, VaEA, and BCE-IBEA on 25, 24, and 17 out of the 26 test problems, respectively. Similarly, for HV, ATM-MOEA/D outperforms AR-MOEA, VaEA, and BCE-IBEA on 21, 19, and 16 out of the 26 test problems, respectively. To gain a visual understanding of the search behaviour of the five algorithms, we selected one representative problem from each category of irregular problems: 3-objective IDTLZ1 for inverted Pareto fronts, CDTLZ2 for high-nonlinear Pareto fronts, 3-objective DTLZ7 for disconnect Pareto fronts, 3-objective DTLZ5 for degenerate Pareto fronts, 3-objective SDTLZ1 for scaled Pareto fronts to understand the search behaviour of the five algorithms. For 3-objective IDTLZ1 (refer to Figure 5), compared with ATM-MOEA/D, solutions obtained by AR-MOEA, VaEA and BCE-IBEA are not uniformly distributed on the Pareto front: VaEA and AR-MOEA has multiple solutions clustered together, while BCE-IBEA has blank regions in its solution set. For the 3-objective CDTLZ2 problem (refer to Figure 6), AR-MOEA generates a limited number of solutions, which can result in sparse coverage of the Pareto front. VaEA also struggles to preserve boundary solutions and may exhibit clustering behavior. Additionally, the uniformity of the solution sets generated by BCE-IBEA is lower compared to ATM-MOEA/D. For 3-objective DTLZ7 (refer to Figure 7), AR-MOEA tends to have multiple solutions clustered closely together, indicating reduced uniformity. Similarly, the solution sets generated by VaEA and BCE-IBEA exhibit inferior uniformity compared to ATM-MOEA/D. For 3-objective DTLZ5 (refer to Figure 7), the uniformity of the solutions obtained by AR-MOEA, VaEA, and BCE-IBEA is inferior to that of ATM-MOEA/D: AR-MOEA and VaEA have gaps in the upper and middle parts of the Pareto front, indicating inferior uniformity. For 3-objective SDTLZ1 (refer to Figure 9), VaEA can not cover the whole Pareto front, leaving significant gaps in the upper corner. Although BCE-IBEA produces solutions that cover the entire Pareto front, the solutions obtained by BCE-IBEA suffer from poor uniformity. ATM-MOEA/D and AR-MOEA have better uniformity than other algorithms in terms of solution distribution. On the other hand, ATM-MOEA/D and AR-MOEA exhibit better uniformity in terms of solution distribution compared to other algorithms. In summary, ATM-MOEA/D outperforms other peer algorithms in terms of IGD and HV on irregular Pareto front problems, as it generates more uniformly distributed solution sets for these problems.



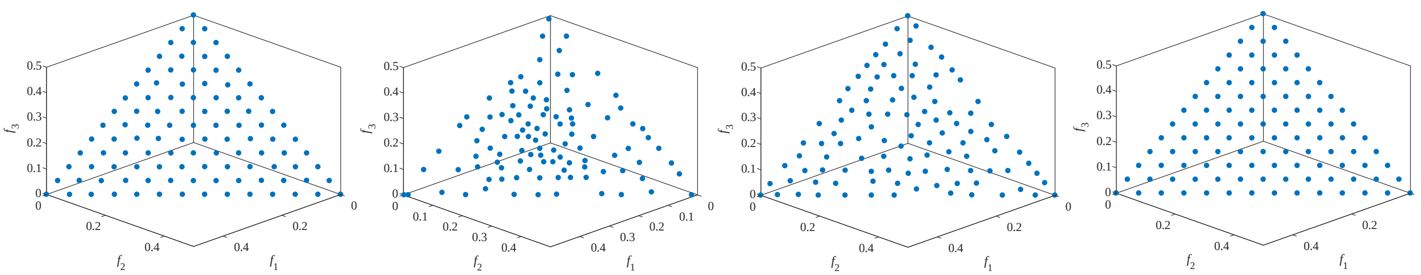

(a) AR-MOEA (b) VaEA (c) BCE-IBEA (d) ATM-MOEA/D

Fig. 1. The final solution set of the four algorithms on 2-objective DTLZ1.



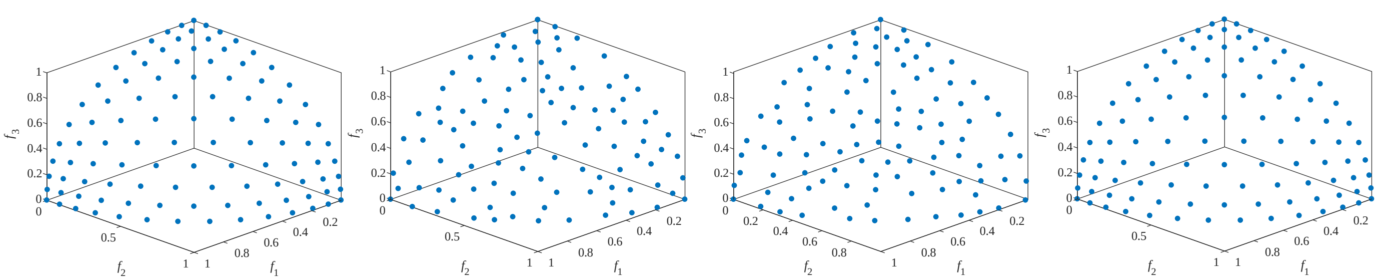

(a) AR-MOEA (b) VaEA (c) BCE-IBEA (d) ATM-MOEA/D

Fig. 2. The final solution set of the four algorithms on ZDT6.



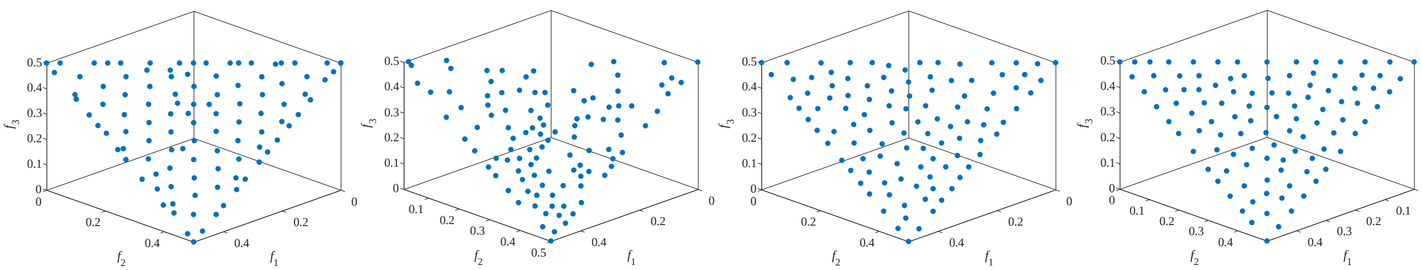

(a) AR-MOEA (b) VaEA (c) BCE-IBEA (d) ATM-MOEA/D

Fig. 3. The final solution set of the four algorithms on 3-objective DTLZ1.



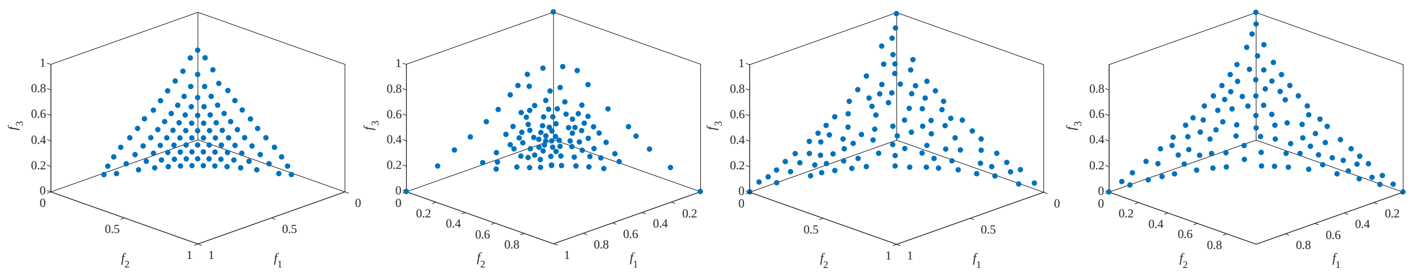

(a) AR-MOEA (b) VaEA (c) BCE-IBEA (d) ATM-MOEA/D

Fig. 4. The final solution set of the four algorithms on 3-objective DTLZ2.



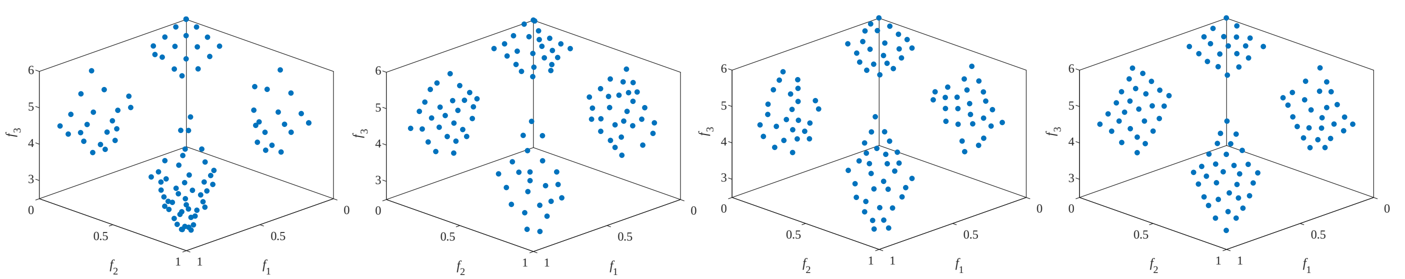

(a) AR-MOEA (b) VaEA (c) BCE-IBEA (d) ATM-MOEA/D

Fig. 5. The final solution set of the four algorithms on the 3-objective IDTLZ1.



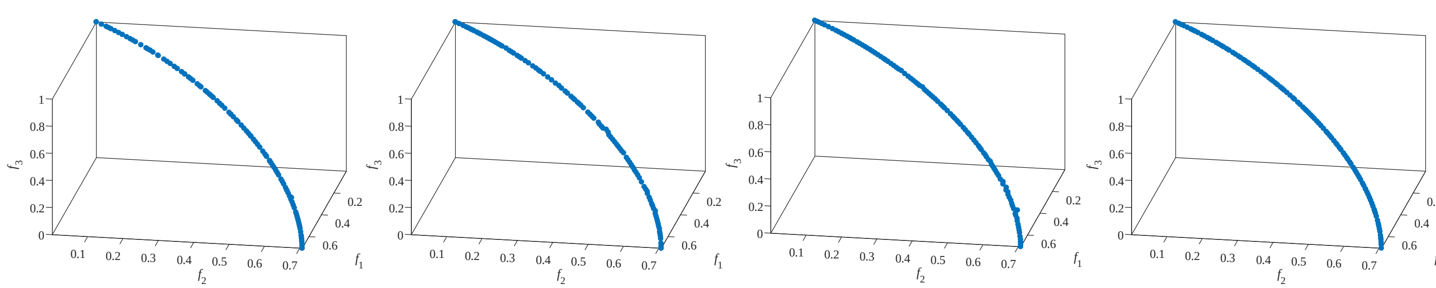

(a) AR-MOEA (b) VaEA (c) BCE-IBEA (d) ATM-MOEA/D

Fig. 6. The final solution set of the four algorithms on 3-objective CDTLZ2.



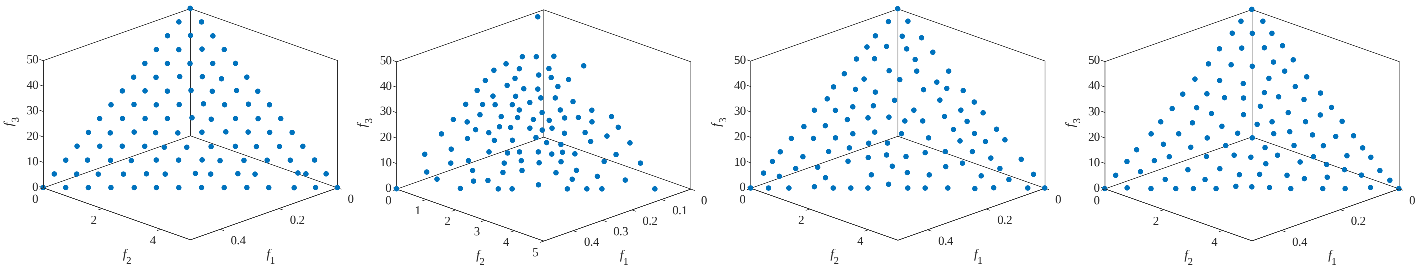

(a) AR-MOEA (b) VaEA (c) BCE-IBEA (d) ATM-MOEA/D

Fig. 7. The final solution set of the four algorithms on the 3-objective DTLZ7.




(a) AR-MOEA (b) VaEA (c) BCE-IBEA (d) ATM-MOEA/D

Fig. 8. The final solution set of the four algorithms on the 3-objective DTLZ5.




(a) AR-MOEA (b) VaEA (c) BCE-IBEA (d) ATM-MOEA/D

Fig. 9. The final solution set of the four algorithms on 3-objective SDTLZ1.

TABLE I
IGD RESULTS (MEAN AND SD) OF THE FOUR ALGORITHMS.

| Pareto front | Problem | AR-MOEA | VaEA | BCE-IBEA | ATM-MOEAD |
|---|---|---|---|---|---|
| Regular | DTLZ1-2 | 2.130E-03(4.7E-04)† | 3.145E-03(2.4E-03)† | 2.099E-03(2.6E-04)† | **1.798E-03(2.8E-05)** |
| | DTLZ2-2 | 4.141E-03(5.4E-06)† | 4.489E-03(5.2E-05)† | 4.189E-03(3.8E-05)† | **4.133E-03(3.8E-08)** |
| | DTLZ4-2 | 1.030E-01(2.6E-01)† | 7.866E-02(2.3E-01)† | 3.256E-01(3.7E-01)† | **4.135E-03(1.2E-05)** |
| | DTLZ1-3 | 1.880E-02(1.3E-04)† | 3.539E-02(2.2E-02)† | 1.923E-02(2.4E-04)† | **1.873E-02(1.0E-04)** |
| | DTLZ2-3 | 5.017E-02(6.8E-05) | 5.282E-02(9.6E-04)† | 5.238E-02(1.1E-03)† | 5.025E-02(3.3E-04) |
| | DTLZ4-3 | 3.302E-01(3.7E-01) | 5.289E-02(6.9E-04)† | 5.320E-02(7.9E-04)† | **5.031E-02(4.7E-04)** |
| | DTLZ1-5 | 5.210E-02(1.1E-04)† | 1.006E-01(2.9E-02)† | **5.172E-02(9.7E-04)**† | 5.205E-02(1.7E-04) |
| | DTLZ2-5 | 1.336E-01(3.7E-04)† | 1.459E-01(2.1E-03)† | 1.448E-01(7.4E-03)† | **1.332E-01(1.1E-04)** |
| | ZDT2 | **3.843E-03(2.2E-05)**† | 4.257E-03(7.2E-05)† | 3.934E-03(4.5E-05)† | 3.848E-03(6.1E-05) |
| | ZDT6 | **3.069E-03(6.7E-05)**† | 3.614E-03(4.1E-04)† | 3.108E-03(1.7E-05) | 3.112E-03(1.1E-05) |
| Irregular | DTLZ5-3 | 4.685E-03(1.2E-04)† | 4.725E-03(1.8E-04)† | 4.252E-03(9.2E-05)† | **4.015E-03(3.9E-05)** |
| | DTLZ7-3 | 1.806E-01(1.8E-01)† | 5.893E-02(1.1E-03)† | 9.304E-02(1.0E-01)† | **5.356E-02(6.2E-04)** |
| | CDTLZ2-3 | 4.515E-02(2.0E-04)† | 6.492E-02(6.2E-03)† | 3.149E-02(3.8E-03)† | **2.848E-02(8.6E-04)** |
| | IDTLZ1-3 | 2.051E-02(8.0E-04)† | 2.766E-02(9.1E-03)† | 1.933E-02(2.6E-04) | **1.931E-02(2.2E-04)** |
| | IDTLZ2-3 | 7.098E-02(2.4E-04)† | 6.812E-02(4.4E-03)† | 5.227E-02(1.9E-03)† | **5.085E-02(4.9E-04)** |
| | ZDT3 | 1.121E-02(1.1E-02)† | 7.875E-03(7.3E-03)† | 7.542E-03(9.0E-03) | **4.649E-03(7.0E-05)** |
| | FON1 | 4.601E-03(1.9E-05)† | 4.940E-03(8.2E-05)† | 4.607E-03(4.9E-05)† | **4.585E-03(3.6E-06)** |
| | SCH1 | 4.790E-02(2.2E-04)† | 5.913E-02(9.9E-03)† | **1.670E-02(1.2E-04)**† | 1.720E-02(2.0E-04) |
| | SCH2 | 1.857E+00(8.1E-03)† | 1.861E+00(8.6E-03)† | 1.765E+00(3.2E-01)† | **2.134E-02(2.1E-04)** |
| | SDTLZ1-3 | 9.161E-01(1.3E-02)† | 1.080E+00(7.2E-01)† | 6.418E-01(3.6E-02)† | **6.187E-01(3.1E-02)** |
| | SDTLZ2-3 | 1.492E+00(7.6E-03)† | 1.521E+00(9.2E-02)† | 1.479E+00(5.5E-02)† | **1.237E+00(4.1E-02)** |
| | VNT2 | 2.089E-02(1.1E-03)† | 2.191E-02(2.1E-03)† | 1.198E-02(2.5E-04)† | **1.106E-02(1.7E-04)** |
| | IDTLZ1-10 | 1.242E-01(7.2E-03)† | 1.111E-01(7.2E-03)† | 1.558E-01(2.8E-02)† | **1.074E-01(8.4E-03)** |
| | MaF1-3 | 4.157E-02(4.3E-04)† | 4.108E-02(5.9E-04)† | 4.030E-02(3.6E-04)† | **3.931E-02(1.7E-04)** |
| | MaF2-3 | 2.969E-02(8.0E-04)† | 2.866E-02(3.0E-04)† | **2.802E-02(3.5E-04)**† | 3.446E-02(7.8E-04) |
| | MaF3-3 | 5.118E-02(1.4E-02)† | 1.562E-01(4.1E-01)† | 5.333E-02(2.7E-02)† | **3.432E-02(2.5E-03)** |
| | MaF4-3 | 3.220E-01(1.3E-02)† | 4.126E-01(1.6E-01)† | 2.752E-01(7.5E-02)† | **2.538E-01(6.3E-02)** |
| | MaF5-3 | 1.270E+00(1.4E+00)† | **2.468E-01(3.8E-03)**† | 2.478E-01(6.2E-03)† | 2.790E-01(2.3E-01) |
| | MaF6-3 | 4.333E-03(7.0E-05)† | 4.429E-03(1.3E-04)† | 3.997E-03(3.4E-05) | 3.997E-03(3.4E-05) |
| | MaF7-3 | 1.870E-01(2.1E-01)† | 5.911E-02(1.4E-03)† | 1.033E-01(1.1E-01)† | **5.401E-02(8.3E-04)** |
| | IMOP1 | 1.035E-01(1.1E-03)† | 1.085E-01(4.7E-03)† | 5.958E-03(4.8E-04)† | **5.338E-03(3.8E-04)** |
| | IMOP2 | 2.076E-01(1.9E-01)† | 3.415E-02(4.6E-02)† | 1.448E-02(4.9E-03)† | **1.175E-02(5.0E-03)** |
| | IMOP3 | 1.848E-02(1.1E-02)† | 3.587E-02(2.5E-02)† | **3.543E-03(1.1E-04)**† | 3.743E-03(2.4E-04) |
| | IMOP4 | 7.904E-03(3.5E-04)† | 7.871E-03(5.6E-04)† | 6.732E-03(8.7E-05) | **6.714E-03(1.1E-04)** |
| | IMOP5 | 3.447E-02(3.8E-04)† | 3.324E-02(5.3E-04)† | 3.264E-02(4.8E-04)† | **3.187E-02(6.6E-04)** |
| | IMOP6 | 6.608E-02(1.3E-01)† | 3.161E-02(5.4E-04)† | 3.000E-02(3.4E-04) | **2.991E-02(2.2E-04)** |
| $+/=/-$ | | 31/2/3 | 34/0/2 | 25/6/5 | |

′†′ indicates that ATM-MOEA/D is of statistically significant difference from the corresponding peer algorithm at a 0.05 level by Wilcoxon's rank sum test. The best mean for each case is highlighted in boldface. The symbols $+, =,$ and $-$ indicate that the results of ATM-MOEA/D are significantly better than, worse than, and equivalent to the corresponding peer algorithm.

TABLE II
HV RESULTS (MEAN AND SD) OF THE FOUR ALGORITHMS.

| Pareto front | Problem | AR-MOEA | VaEA | BCE-IBEA | ATM-MOEAD |
|---|---|---|---|---|---|
| Regular | DTLZ1-2 | 8.732E-01(5.4E-04)† | 8.697E-01(9.2E-03)† | 8.732E-01(3.4E-04)† | **8.737E-01(1.5E-04)** |
| | DTLZ2-2 | 3.210E+00(1.2E-05)† | 3.210E+00(1.7E-04)† | **3.210E+00(1.2E-04)**† | 3.210E+00(1.6E-07) |
| | DTLZ4-2 | 3.049E+00(4.2E-01)† | 3.089E+00(3.7E-01)† | 2.686E+00(6.1E-01) | **3.210E+00(7.0E-05)** |
| | DTLZ1-3 | 9.739E-01(1.6E-04)† | 9.643E-01(1.1E-02)† | 9.738E-01(1.2E-04)† | **9.741E-01(3.1E-05)** |
| | DTLZ2-3 | **7.418E+00(9.7E-05)**† | 7.412E+00(1.9E-03)† | 7.413E+00(3.9E-03)† | 7.418E+00(2.7E-03) |
| | DTLZ4-3 | 6.663E+00(1.2E+00)† | 7.411E+00(1.9E-03)† | 7.412E+00(3.8E-03)† | **7.418E+00(3.2E-03)** |
| | DTLZ1-5 | 9.990E-01(5.6E-06)† | 9.922E-01(5.7E-03)† | 9.989E-01(4.1E-05)† | **9.990E-01(5.0E-06)** |
| | DTLZ2-5 | 3.170E+01(1.4E-04)† | 3.167E+01(4.0E-03)† | 3.168E+01(6.0E-03)† | **3.170E+01(9.1E-05)** |
| | ZDT2 | 3.328E+00(2.3E-04)† | 3.327E+00(2.8E-04)† | **3.328E+00(8.9E-04)**† | 3.328E+00(2.5E-03) |
| | ZDT6 | 3.041E+00(6.3E-04) | 3.040E+00(1.9E-03)† | **3.042E+00(6.7E-05)**† | 3.040E+00(3.8E-03) |
| Irregular | DTLZ5-3 | 6.101E+00(3.3E-04)† | 6.101E+00(2.0E-03)† | 6.105E+00(1.1E-04)† | **6.103E+00(1.1E-04)** |
| | DTLZ7-3 | 1.222E+01(1.8E+00)† | 1.338E+01(3.4E-02)† | 1.303E+01(1.1E+00)† | **1.351E+01(9.4E-03)** |
| | CDTLZ2-3 | 7.946E+00(6.1E-05)† | 7.940E+00(1.9E-03)† | 7.950E+00(4.5E-04)† | **7.951E+00(1.8E-04)** |
| | IDTLZ1-3 | 6.867E-01(3.6E-03) | 6.695E-01(1.3E-02)† | 6.868E-01(2.1E-03)† | **6.877E-01(9.1E-04)** |
| | IDTLZ2-3 | 6.655E+00(1.4E-03)† | 6.657E+00(1.3E-02)† | 6.711E+00(4.7E-03)† | **6.729E+00(3.7E-03)** |
| | ZDT3 | 4.752E+00(1.4E-01)† | 4.787E+00(9.3E-02)† | 4.778E+00(1.1E-01)† | **4.815E+00(4.2E-05)** |
| | FON1 | 3.062E+00(2.5E-04)† | 3.062E+00(2.5E-04)† | **3.062E+00(7.8E-04)**† | 3.062E+00(3.3E-05) |
| | SCH1 | 2.225E+01(2.0E-04)† | 2.222E+01(1.6E-02)† | **2.227E+01(8.8E-04)**† | 2.227E+01(1.1E-03) |
| | SCH2 | 3.465E+01(1.8E-02)† | 3.464E+01(2.0E-02)† | 3.479E+01(5.0E-01)† | **3.825E+01(2.1E-03)** |
| | SDTLZ1-3 | **1.403E+02(1.5E-01)** | 1.350E+02(5.4E+00)† | 1.402E+02(8.8E-02)† | 1.403E+02(7.7E-02) |
| | SDTLZ2-3 | 7.485E+02(3.3E-01)† | 7.429E+02(2.1E+00)† | 7.460E+02(1.5E+00)† | **7.489E+02(8.8E-01)** |
| | VNT2 | 1.647E+03(9.4E-01)† | 1.646E+03(2.0E+00)† | 1.648E+03(1.7E-02)† | **1.648E+03(8.6E-03)** |
| | IDTLZ1-10 | **4.824E-07(6.8E-07)** | 4.168E-07(6.6E-07) | 1.412E-07(2.4E-07) | 2.625E-07(3.8E-07) |
| | MaF1-3 | 1.276E-01(5.2E-04)† | 1.266E-01(8.4E-04)† | 1.278E-01(5.2E-04)† | **1.299E-01(2.9E-04)** |
| | MaF2-3 | 2.621E-01(6.8E-04) | 2.632E-01(8.6E-04)† | **2.648E-01(7.7E-04)**† | 2.621E-01(4.8E-04) |
| | MaF3-3 | 9.366E-01(1.4E-02)† | 8.812E-01(1.7E-01)† | 9.327E-01(2.3E-02)† | **9.501E-01(1.2E-03)** |
| | MaF4-3 | 2.779E+01(6.0E-01)† | 2.703E+01(2.6E+00)† | 2.802E+01(2.0E+00)† | **2.917E+01(6.8E-01)** |
| | MaF5-3 | 1.916E+01(9.1E+00)† | 2.631E+01(1.1E-01)† | **2.656E+01(7.6E-02)**† | 2.638E+01(2.4E+00) |
| | MaF6-3 | 8.274E-02(4.4E-05)† | 8.279E-02(5.7E-05)† | 8.298E-02(2.9E-05)† | **8.301E-02(2.0E-05)** |
| | MaF7-3 | 1.223E+01(1.9E+00)† | 1.338E+01(4.4E-02)† | 1.292E+01(1.2E+00)† | **1.351E+01(5.5E-03)** |
| | IMOP1 | 9.820E-01(4.3E-05)† | 9.811E-01(6.6E-04)† | **9.846E-01(6.5E-05)** | 9.846E-01(6.3E-05) |
| | IMOP2 | 4.308E-02(2.8E-02)† | 6.848E-02(6.1E-03)† | **7.071E-02(8.5E-05)**† | 7.070E-02(5.1E-04) |
| | IMOP3 | 1.106E+00(8.1E-03)† | 1.076E+00(4.9E-02)† | **1.116E+00(6.8E-05)** | 1.115E+00(1.5E-03) |
| | IMOP4 | **3.442E-01(2.7E-04)** | 3.441E-01(4.4E-04) | 3.440E-01(1.9E-04)† | 3.441E-01(2.4E-04) |
| | IMOP5 | 8.223E-01(9.6E-04)† | 8.236E-01(1.6E-03)† | **8.250E-01(1.9E-03)** | 8.247E-01(3.0E-03) |
| | IMOP6 | 4.182E-01(4.7E-02)† | 4.283E-01(9.1E-04)† | 4.296E-01(9.7E-04)† | **4.324E-01(5.5E-04)** |
| $+/=/-$ | | 28/6/2 | 29/3/4 | 22/5/9 | |

′†′ indicates that ATM-MOEA/D is of statistically significant difference from the corresponding peer algorithm at a 0.05 level by Wilcoxon's rank sum test. The best mean for each case is highlighted in boldface. The symbols $+$, $=$, and $-$ indicate that the results of ATM-MOEA/D are significantly better than, worse than, and equivalent to the corresponding peer algorithm.